

Introduction

The **Travelling Salesman Problem (TSP)** is a combinatorial optimisation problem that aims to **determine the optimal service sequence and route** from a depot to geographically dispersed customers, starting and ending at the depot, while considering operational constraints. The main objective is to find the most efficient Hamiltonian cycle, a path that visits each customer exactly once.

The TSP non-polynomial complexity (**NP-Hard**) makes it computationally challenging to find solutions for larger problem instances.

A range of heuristic and metaheuristic algorithms have emerged to address these challenges. **It remains an integral challenge to dynamically and efficiently search and evaluate the solution space.**

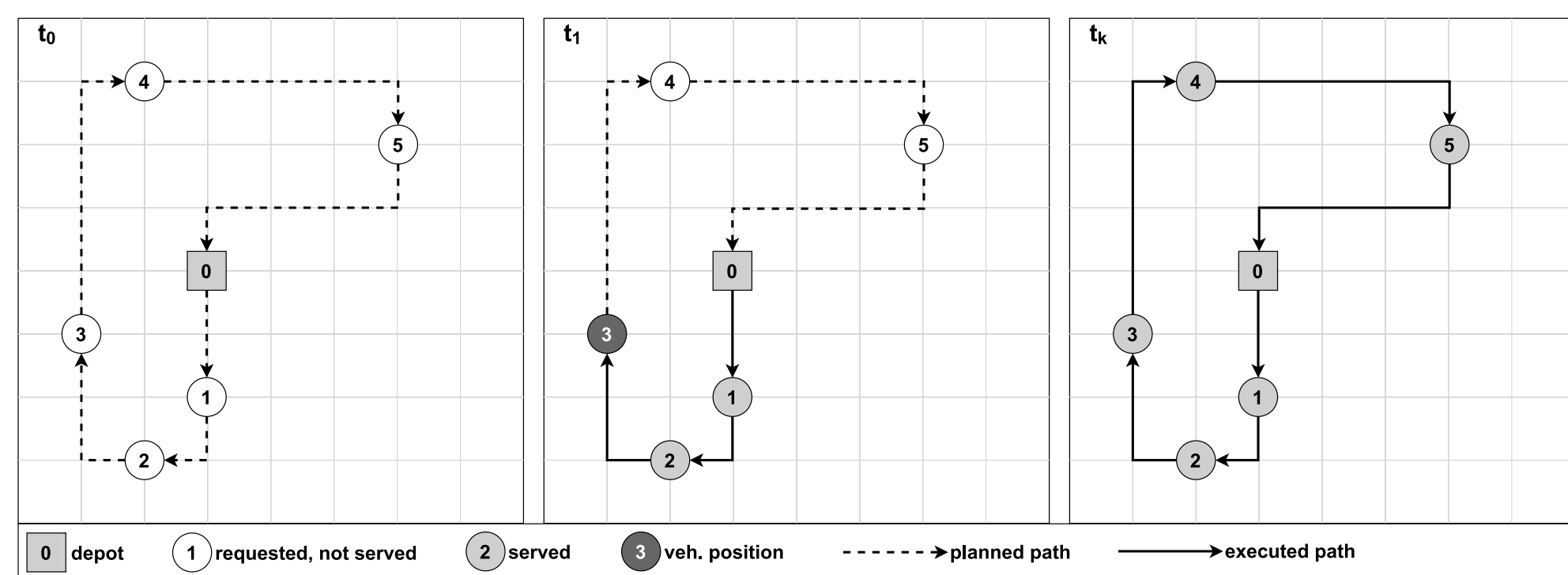


Figure 1. Route planning and execution of a single vehicle

The **Genetic Algorithm (GA)** is a metaheuristic population-based optimisation method inspired by the principle of natural selection in evolutionary biology. It involves the **iterative generation, evaluation, and evolution of a population of solutions over multiple generations** until a termination criterion is met. What sets the GA apart is its ability to explore a vast and diverse solution space, increasing the likelihood of escaping local optima and finding the global optimal solution.

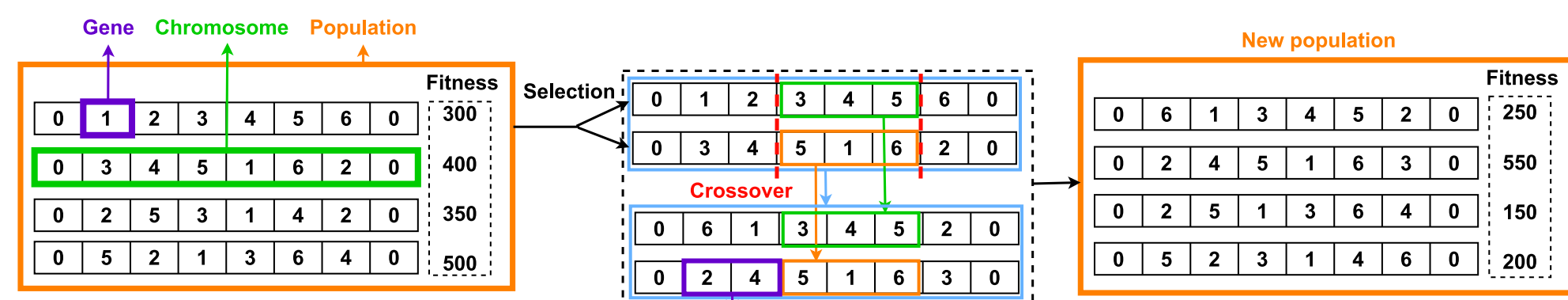


Figure 2. TSP Genetic algorithm

Genetic Algorithm Operators

Selection operators play a crucial role in choosing the most promising individuals from the current generation for reproduction and elimination, ensuring the quality of the population evolves over time.

Crossover operators recombine genetic information from two parent solutions to produce one or more offspring solutions.

Mutation operators introduce occasional, random variations in an individual's genes. This helps maintain population diversity and prevents premature convergence to suboptimal solutions.

Effective and dynamic utilisation of operators is vital for maintaining a diverse population and guiding the search towards the global optimum.

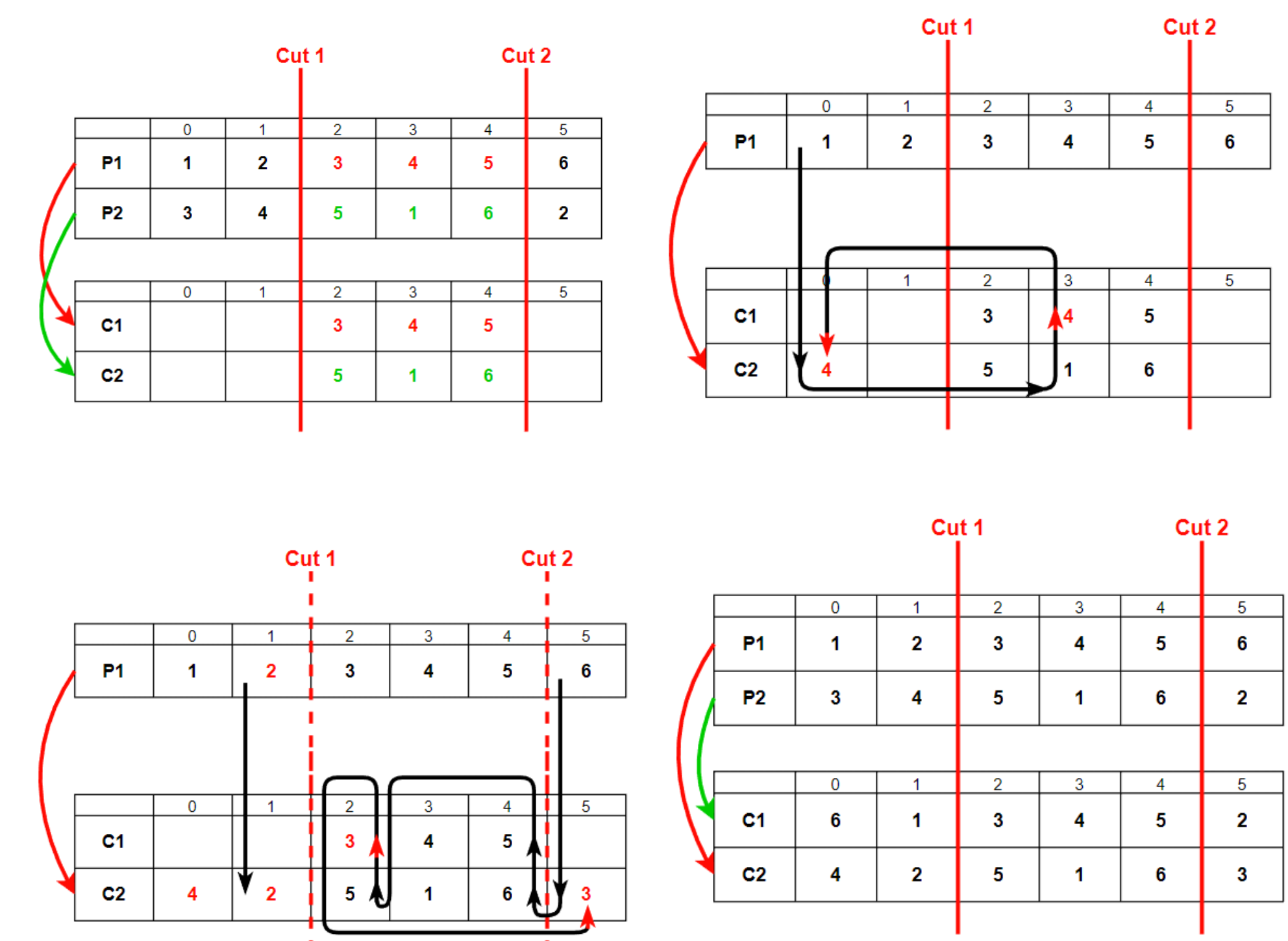


Figure 3. Partially Mapped Crossover (PMX) example

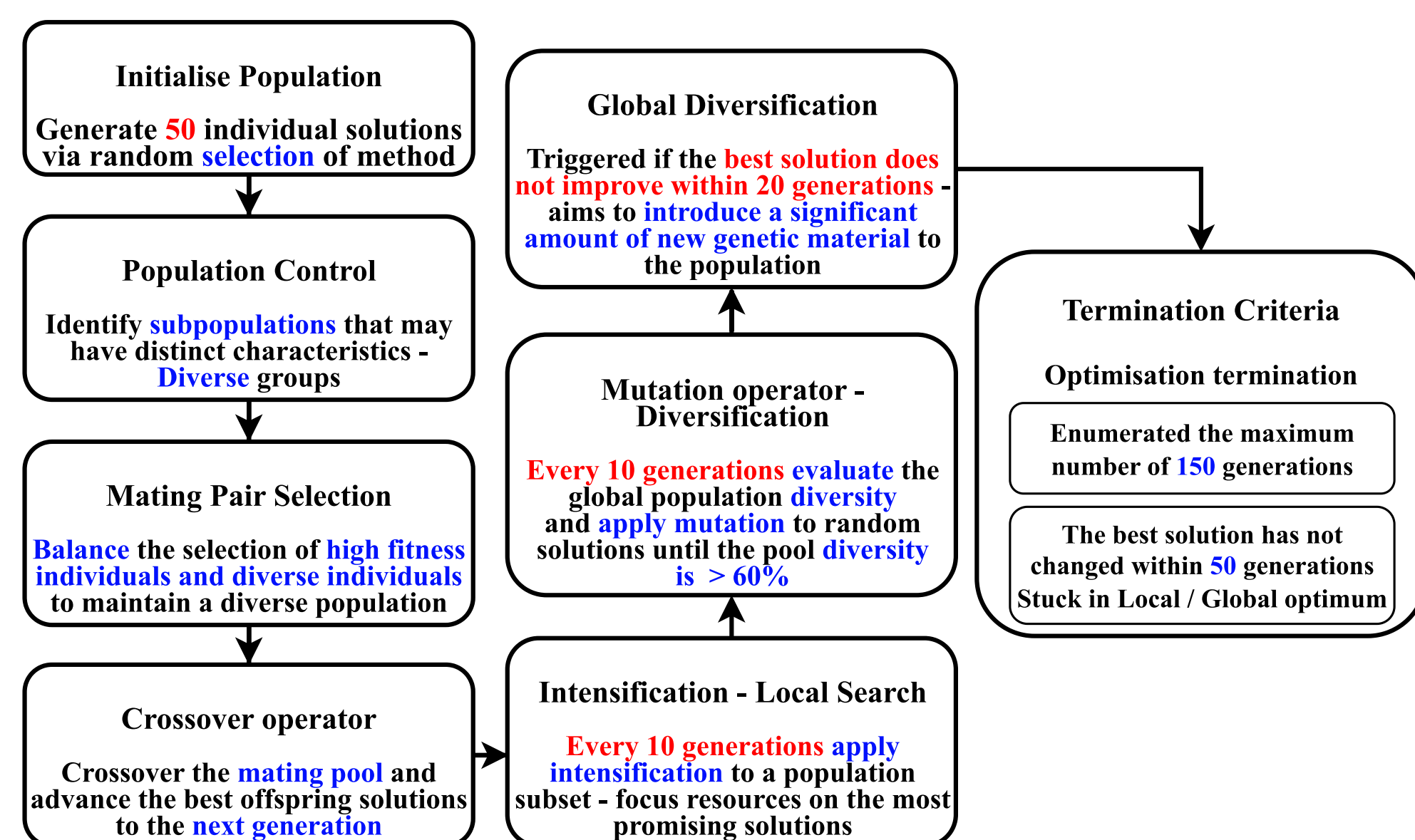
Population-Control Genetic Algorithm (PC-GA)

This research introduces a **Population-Control Genetic Algorithm (PC-GA)** tailored for solving the TSP, emphasising enhanced population diversity and search efficiency by maintaining genetic diversity and preventing premature convergence.

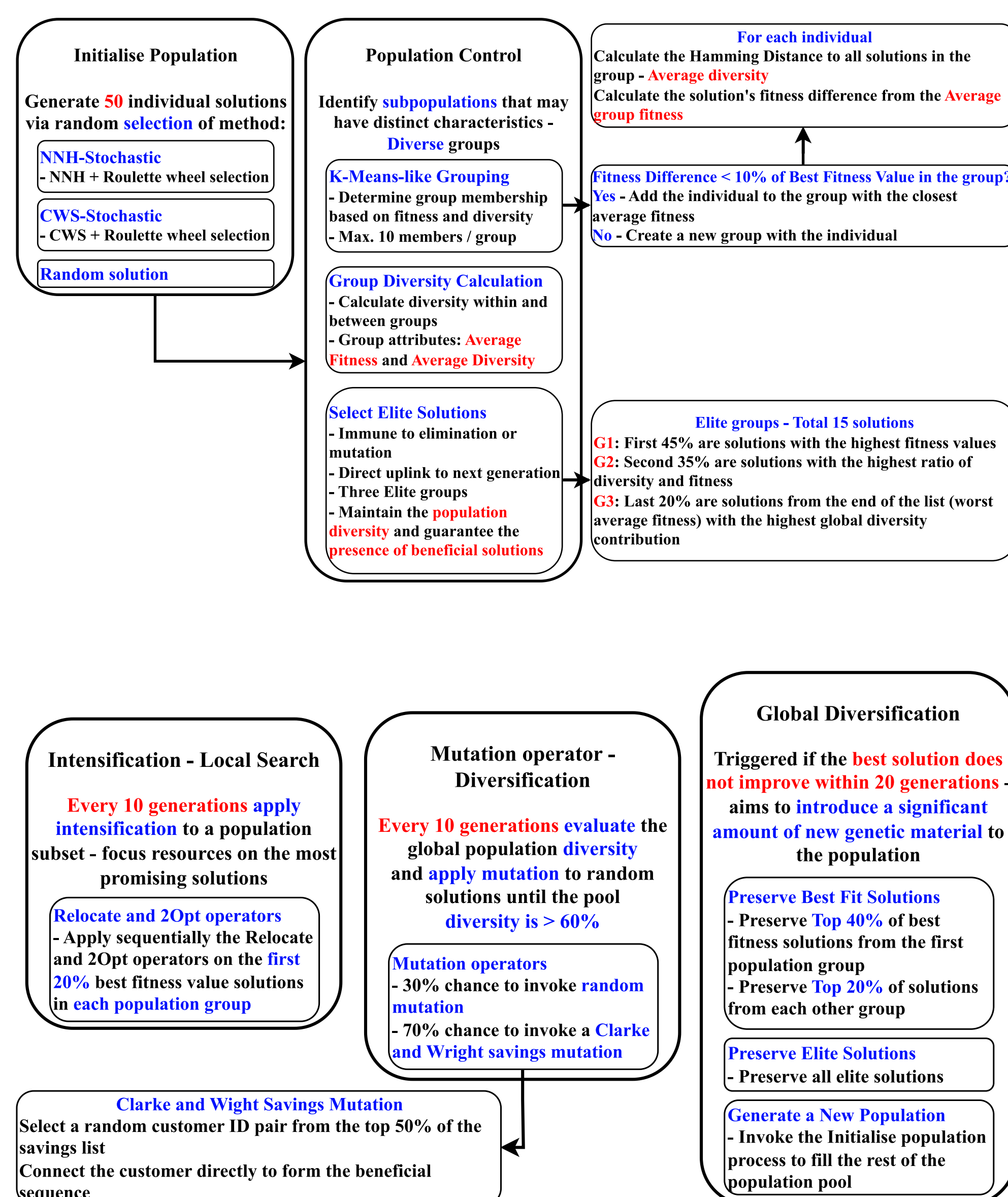
The **initial population consists of 50 solutions** generated via three methods: nearest-neighbour heuristic combined with stochastic roulette wheel selection, stochastic Clarke and Wright savings, and random solution generation, ensuring extensive initial diversity. To maintain diversity, **the algorithm groups solutions based on fitness and a diversity metric computed using Hamming distances**. Groups hold up to 10 solutions each, structured around relative fitness. The diversity contribution of each solution is calculated using average pairwise distances to other solutions, ensuring the preservation of genetic diversity.

The selection mechanism retains elite individuals (top 15 solutions), with 45% based solely on fitness, 35% balancing fitness and diversity, and the remaining 20% explicitly selected for diversity contributions. Crossover pairs are strategically chosen from diverse groups based on combined fitness and diversity, utilising **Partially Mapped Crossover (PMX)** to ensure offspring feasibility. **Mutation** maintains diversity through random (30%) or Clarke and Wright savings-based changes (70%), **applied until the population surpasses a diversity threshold (60% of the theoretical maximum)**. Regular local searches, such as **Intra-Relocate and Intra-2-Opt**, refine solutions.

A global diversification process is initiated if the best solution stagnates over time. In this process, the top 40% of solutions from the first group, along with 20% from each of the other groups, are retained. The complete set of elite solutions is also preserved, and the remaining slots are filled with new, randomly generated individuals to reintroduce diversity into the population. The optimisation process ends when either the **maximum number of generations (150 generations)** is reached or when the **best solution remains unchanged for 50 generations**, indicating that the algorithm has converged.



Components of the flow chart



Results

The **Population-Control Genetic Algorithm (PC-GA)** was compared to a classic, random operator utilisation GA and a series of heuristic algorithms: Clarke and Wright Savings (CWS), Nearest Insertion Heuristic (NIH), Farthest Insertion Heuristic (FIH), and Nearest Neighbour Heuristic (NNH).

The algorithms were analysed on **TSPLIB** benchmark instances: berlin52, pr76, and lin105. All algorithms were implemented and tested in C#. The best result in each column is highlighted using red font. Each GA was run five consecutive times on each TSPLIB instance.

Table 1. Instance: berlin52; Dimension: 52; Optimal: 7542

Algorithm	Initial solution	Execution time [s]	Local Search	Divergence from optimum [%]
CWS	14005	0,0117069	7712	2,25%
NIH	8647	2,950103	8121	7,68%
FIH	8499	0,1377876	7842	3,98%
NNH	9007	0,002527	8011	6,22%
GA	Min 9820	1,7778259	8251	9,4%
	Max 11197	1,8158727	8249	9,37%
	Avg 10580	1,7990394	8032	6,5%
PC-GA	Min 7570	7,4732371	7570	0,37%
	Max 7737	7,3911067	7737	2,59%
	Avg 7603	8,1810017	7603	0,81%

Table 2. Instance: pr76; Dimension: 76; Optimal: 108 159

Algorithm	Initial solution	Execution time [s]	Local Search	Divergence from optimum [%]
CWS	210 271	0,0146047	111 803	3,37%
NIH	118 381	16,6086624	116 183	7,42%
FIH	117 392	0,5728099	109 658	1,39%
NNH	153 494	0,0034946	109 822	1,54%
GA	Min 149 228	4,0466916	115 338	6,64%
	Max 171 544	4,0678091	116 539	7,75%
	Avg 158 449	4,0761037	114 399	5,77%
PC-GA	Min 108 193	17,1541522	108 193	0,03%
	Max 109 058	18,408982	109 058	0,83%
	Avg 108 567	22,4171955	108 567	0,38%

Table 3. Instance: lin105; Dimension: 105; Optimal: 14 379

Algorithm	Initial solution	Execution time [s]	Local Search	Divergence from optimum [%]
CWS	38 478	0,0270486	15 211	5,79%
NIH	16 253	76,4744554	14 734	2,47%
FIH	15 755	1,9429209	14 793	2,88%
NNH	20 398	0,0098893	16 655	15,83%
GA	Min 20 732	8,1399695	15 513	7,89%
	Max 23 118	8,1686373	14 912	3,71%
	Avg 21 848	8,7990394	15 199	5,7%
PC-GA	Min 14 418	69,4053223	14 418	0,27%
	Max 14 625	54,0597133	14 625	1,71%
	Avg 14 522	61,7364888	14 522	0,99%

Outlook

The developed algorithm efficiently and dynamically guides the search space towards the global optimum, generating optimal or near-optimal results.

The key innovation of the PC-GA lies in its population grouping strategy, where chromosomes are assigned to groups based on their fitness relative to the group's average. This approach promotes diversity within the population, facilitating a more thorough exploration of the solution space and preventing premature convergence.

The main downside of the developed approach is manifested in its complexity, which increases significantly with problem size. This increase stems from the diversity preservation mechanisms, which require extensive pairwise evaluations and mutation processes. While these methods improve solution quality, they introduce significant overhead.

Further research will include examining Electric Vehicle (EV) operational constraints in relation to the problem and expanding the problem to a multi-vehicle environment.

Acknowledgments

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